6.1 Exercise Set

FOR EXTRA HELP

Concept Reinforcement *In each of Exercises 1–6, match the function described with the appropriate domain from the column on the right. Some choices of domain will not be used.*

1. (e) $f(x) = \dfrac{2 - x}{x - 5}$
2. (c) $f(x) = \dfrac{x + 5}{x + 2}$
3. (i) $g(x) = \dfrac{x - 3}{(x - 2)(x - 5)}$
4. (f) $g(x) = \dfrac{x + 3}{(x + 2)(x - 5)}$
5. (d) $h(x) = \dfrac{(x - 2)(x - 3)}{x + 3}$
6. (b) $f(x) = \dfrac{(x + 2)(x + 3)}{x - 3}$

a) $\{x \mid x \neq -5, x \neq 2\}$
b) $\{x \mid x \neq 3\}$
c) $\{x \mid x \neq -2\}$
d) $\{x \mid x \neq -3\}$
e) $\{x \mid x \neq 5\}$
f) $\{x \mid x \neq -2, x \neq 5\}$
g) $\{x \mid x \neq 2\}$
h) $\{x \mid x \neq -2, x \neq -5\}$
i) $\{x \mid x \neq 2, x \neq 5\}$
j) $\{x \mid x \neq -5\}$

Processing Orders. *Jasmine usually takes 3 hr more than Molly does to process a day's orders at Books To Go. If Molly takes t hr to process a day's orders, the function given by*

$$H(t) = \frac{t^2 + 3t}{2t + 3}$$

can be used to determine how long it would take if they worked together.

7. How long will it take them, working together, to complete a day's orders if Molly can process the orders alone in 5 hr? $\frac{40}{13}$ hr, or $3\frac{1}{13}$ hr
8. How long will it take them, working together, to complete a day's orders if Molly can process the orders alone in 7 hr? $\frac{70}{17}$ hr, or $4\frac{2}{17}$ hr

For each rational function, find the function values indicated, provided the value exists.

9. $v(t) = \dfrac{4t^2 - 5t + 2}{t + 3}$; $v(0), v(-2), v(7)$ $\frac{2}{3}$; 28; $\frac{163}{10}$
10. $f(x) = \dfrac{5x^2 + 4x - 12}{6 - x}$; $f(0), f(-1), f(3)$ -2; $-\frac{11}{7}$; 15
11. $g(x) = \dfrac{2x^3 - 9}{x^2 - 4x + 4}$; $g(0), g(2), g(-1)$ $-\frac{9}{4}$; does not exist; $-\frac{11}{9}$
12. $r(t) = \dfrac{t^2 - 5t + 4}{t^2 - 9}$; $r(1), r(2), r(-3)$ 0; $\frac{2}{5}$; does not exist

Find the domain of each function.

13. $f(x) = \dfrac{25}{-7x}$ ⊡
14. $f(x) = \dfrac{14}{-5x}$ ⊡
15. $r(t) = \dfrac{t - 3}{t + 8}$ ⊡
16. $r(a) = \dfrac{a - 8}{a + 7}$ ⊡
17. $f(x) = \dfrac{x^2 - 16}{x^2 - 3x - 28}$ ⊡
18. $f(p) = \dfrac{p^2 - 9}{p^2 - 7p + 10}$ ⊡
19. $g(m) = \dfrac{m^3 - 2m}{m^2 - 25}$ ⊡
20. $g(x) = \dfrac{7 - 3x + x^2}{49 - x^2}$ ⊡

Simplify by removing a factor equal to 1.

21. $\dfrac{15x}{5x^2}$ $\frac{3}{x}$
22. $\dfrac{7a^3}{21a}$ $\frac{a^2}{3}$
23. $\dfrac{18t^3w^2}{27t^7w}$ $\frac{2w}{3t^4}$
24. $\dfrac{8y^5z}{4y^9z^3}$ $\frac{2}{y^4z^2}$

⊡ Answers to Exercises 13–20 are on p. IA-13.

25. $\dfrac{2a-10}{2}$ $a-5$

26. $\dfrac{3a+12}{3}$ $a+4$

27. $\dfrac{5x}{25xy-30x}$ $\dfrac{1}{5y-6}$

28. $\dfrac{21y}{6xy-9y}$ $\dfrac{7}{2x-3}$

29. $\dfrac{20-4x}{3x-15}$ $-\dfrac{4}{3}$

30. $\dfrac{2-x}{7x-14}$ $-\dfrac{1}{7}$

Write simplified form for each of the following. Be sure to list all restrictions on the domain, as in Example 4.

31. $f(x)=\dfrac{5x+30}{x^2+6x}$ ⊡

32. $f(x)=\dfrac{3x+30}{x^2+10x}$ ⊡

33. $g(x)=\dfrac{x^2-9}{5x+15}$ ⊡

34. $g(x)=\dfrac{8x-16}{x^2-4}$ ⊡

35. $h(x)=\dfrac{2-x}{7x-14}$ ⊡

36. $h(x)=\dfrac{4-x}{12x-48}$ ⊡

37. $f(t)=\dfrac{t^2-16}{t^2-8t+16}$ ⊡

38. $f(t)=\dfrac{t^2-25}{t^2+10t+25}$ ⊡

39. $g(t)=\dfrac{21-7t}{3t-9}$ ⊡

40. $g(t)=\dfrac{12-6t}{5t-10}$ ⊡

41. $h(t)=\dfrac{t^2+5t+4}{t^2-8t-9}$ ⊡

42. $h(t)=\dfrac{t^2-3t-4}{t^2+9t+8}$ ⊡

43. $f(x)=\dfrac{9x^2-4}{3x-2}$ ⊡

44. $f(x)=\dfrac{4x^2-1}{2x-1}$ ⊡

45. $g(t)=\dfrac{16-t^2}{t^2-8t+16}$ ⊡

46. $g(p)=\dfrac{25-p^2}{p^2+10p+25}$ ⊡

Multiply and, if possible, simplify.

47. $\dfrac{3y^3}{5z}\cdot\dfrac{10z^4}{7y^6}$ $\dfrac{6z^3}{7y^3}$

48. $\dfrac{20y}{9z^7}\cdot\dfrac{6z^4}{5y^2}$ $\dfrac{8}{3z^3y}$

49. $\dfrac{8x-16}{5x}\cdot\dfrac{x^3}{5x-10}$ $\dfrac{8x^2}{25}$

50. $\dfrac{5t^3}{4t-8}\cdot\dfrac{6t-12}{10t}$ $\dfrac{3t^2}{4}$

51. $\dfrac{y^2-9}{y^2}\cdot\dfrac{y^2-3y}{y^2-y-6}$ $\dfrac{(y+3)(y-3)}{y(y+2)}$

52. $\dfrac{y^2+10y+25}{y^2-9}\cdot\dfrac{y^2+3y}{y+5}$ $\dfrac{y(y+5)}{y-3}$

53. $\dfrac{7a-14}{4-a^2}\cdot\dfrac{5a^2+6a+1}{35a+7}$ $-\dfrac{a+1}{2+a}$

54. $\dfrac{a^2-1}{2-5a}\cdot\dfrac{15a-6}{a^2+5a-6}$ $-\dfrac{3(a+1)}{a+6}$

Aha! **55.** $\dfrac{t^3-4t}{t-t^4}\cdot\dfrac{t^4-t}{4t-t^3}$ 1

⊡ Answers to Exercises 31–46 and 73–76 are on p. IA-13.

56. $\dfrac{x^2-6x+9}{12-4x}\cdot\dfrac{x^6-9x^4}{x^3-3x^2}$ $\dfrac{x^2(x+3)(x-3)}{-4}$

57. $\dfrac{c^3+8}{c^5-4c^3}\cdot\dfrac{c^6-4c^5+4c^4}{c^2-2c+4}$ $c(c-2)$

58. $\dfrac{t^3-27}{t^4-9t^2}\cdot\dfrac{t^5-6t^4+9t^3}{t^2+3t+9}$ $\dfrac{t(t-3)^2}{t+3}$

59. $\dfrac{a^3-b^3}{3a^2+9ab+6b^2}\cdot\dfrac{a^2+2ab+b^2}{a^2-b^2}$ $\dfrac{a^2+ab+b^2}{3(a+2b)}$

60. $\dfrac{x^3+y^3}{x^2+2xy-3y^2}\cdot\dfrac{x^2-y^2}{3x^2+6xy+3y^2}$ $\dfrac{x^2-xy+y^2}{3(x+3y)}$

Divide and, if possible, simplify.

61. $\dfrac{12a^3}{5b^2}\div\dfrac{4a^2}{15b}$ $\dfrac{9a}{b}$

62. $\dfrac{9x^7}{8y}\div\dfrac{15x^2}{4y}$ $\dfrac{3x^5}{10}$

63. $\dfrac{5x+20}{x^6}\div\dfrac{x+4}{x^2}$ $\dfrac{5}{x^4}$

64. $\dfrac{3a+15}{a^9}\div\dfrac{a+5}{a^8}$ $\dfrac{3}{a}$

65. $\dfrac{25x^2-4}{x^2-9}\div\dfrac{2-5x}{x+3}$ $\dfrac{5x+2}{x-3}$

66. $\dfrac{4a^2-1}{a^2-4}\div\dfrac{2a-1}{2-a}$ $-\dfrac{2a+1}{a+2}$

67. $\dfrac{5y-5x}{15y^3}\div\dfrac{x^2-y^2}{3x+3y}$ $-\dfrac{1}{y^3}$

68. $\dfrac{x^2-y^2}{4x+4y}\div\dfrac{3y-3x}{12x^2}$ $-x^2$

69. $\dfrac{y^2-36}{y^2-8y+16}\div\dfrac{3y-18}{y^2-y-12}$ $\dfrac{(y+6)(y+3)}{3(y-4)}$

70. $\dfrac{x^2-16}{x^2-10x+25}\div\dfrac{3x-12}{x^2-3x-10}$ $\dfrac{(x+4)(x+2)}{3(x-5)}$

71. $\dfrac{x^3-64}{x^3+64}\div\dfrac{x^2-16}{x^2-4x+16}$ $\dfrac{x^2+4x+16}{(x+4)^2}$

72. $\dfrac{8y^3-27}{64y^3-1}\div\dfrac{4y^2-9}{16y^2+4y+1}$ $\dfrac{4y^2+6y+9}{(4y-1)(2y+3)}$

Write simplified form for each of the following. Be sure to list all restrictions on the domain.

73. $f(t)=\dfrac{t^2-100}{5t+20}\cdot\dfrac{t+4}{t-10}$ ⊡

74. $g(n)=\dfrac{n+5}{n-5}\cdot\dfrac{n^2-25}{2n+2}$ ⊡

75. $g(x)=\dfrac{x^2-2x-35}{2x^3-3x^2}\cdot\dfrac{4x^3-9x}{7x-49}$ ⊡

76. $h(t)=\dfrac{t^2-10t+9}{t^2-1}\cdot\dfrac{1-t^2}{t^2-5t-36}$ ⊡

77. $f(x) = \dfrac{x^2 - 4}{x^3} \div \dfrac{x^5 - 2x^4}{x + 4}$ ⊡

78. $g(x) = \dfrac{x^2 - 9}{x^2} \div \dfrac{x^5 + 3x^4}{x + 2}$ ⊡

79. $h(n) = \dfrac{n^3 + 3n}{n^2 - 9} \div \dfrac{n^2 + 5n - 14}{n^2 + 4n - 21}$ ⊡

80. $f(x) = \dfrac{x^3 + 4x}{x^2 - 16} \div \dfrac{x^2 + 8x + 15}{x^2 + x - 20}$ ⊡

Perform the indicated operations and, if possible, simplify. Recall that multiplications and divisions are performed in order from left to right.

81. $\dfrac{4x^2 - 9y^2}{8x^3 - 27y^3} \div \dfrac{4x + 6y}{3x - 9y} \cdot \dfrac{4x^2 + 6xy + 9y^2}{4x^2 - 8xy + 3y^2}$ ⊡

82. $\dfrac{5x^2 - 5y^2}{27x^3 + 8y^3} \div \dfrac{x^2 - 2xy + y^2}{9x^2 - 6xy + 4y^2} \cdot \dfrac{6x + 4y}{10x - 15y}$ ⊡

83. $\dfrac{a^3 - ab^2}{2a^2 + 3ab + b^2} \cdot \dfrac{4a^2 - b^2}{a^2 - 2ab + b^2} \div \dfrac{a^2 + a}{a - 1}$ ⊡

84. $\dfrac{2x + 4y}{2x^2 + 5xy + 2y^2} \cdot \dfrac{4x^2 - y^2}{8x^2 - 8} \div \dfrac{x^2 + 4xy + 4y^2}{x^2 - 6xy + 9y^2}$ ⊡

Determine the vertical asymptotes of the graph of each function.

85. $f(x) = \dfrac{3x - 12}{3x + 15}$ $x = -5$

86. $f(x) = \dfrac{4x - 20}{4x + 12}$ $x = -3$

87. $g(x) = \dfrac{12 - 6x}{5x - 10}$ No vertical asymptotes

88. $r(x) = \dfrac{21 - 7x}{3x - 9}$ No vertical asymptotes

89. $t(x) = \dfrac{x^3 + 3x^2}{x^2 + 6x + 9}$ $x = -3$

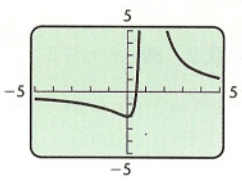
90. $g(x) = \dfrac{x^2 - 4}{2x^2 - 5x + 2}$ $x = \frac{1}{2}$

91. $f(x) = \dfrac{x^2 - x - 6}{x^2 - 6x + 8}$ $x = 2, x = 4$

92. $f(x) = \dfrac{x^2 + 2x + 1}{x^2 - 2x + 1}$ $x = 1$

In Exercises 93–98, match each function with one of the following graphs.

a)



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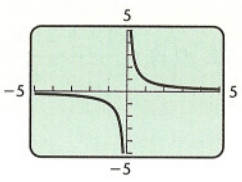

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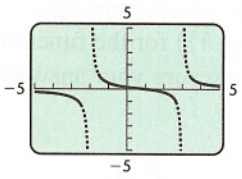

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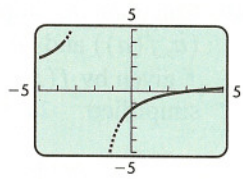

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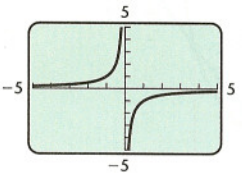
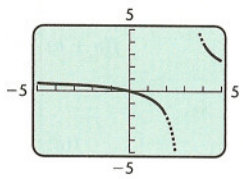


f)



93. $h(x) = \dfrac{1}{x}$ (b)

94. $q(x) = -\dfrac{1}{x}$ (e)

95. $f(x) = \dfrac{x}{x - 3}$ (f)

96. $g(x) = \dfrac{x - 3}{x + 2}$ (d)

97. $r(x) = \dfrac{4x - 2}{x^2 - 2x + 1}$ (a)

98. $t(x) = \dfrac{x - 1}{x^2 - x - 6}$ (c)

TW **99.** Explain why the graphs of $f(x) = 5x$ and $g(x) = \dfrac{5x^2}{x}$ differ.

TW **100.** If a rational expression is undefined for $x = 5$ and $x = -3$, what is the degree of the denominator? Why?

SKILL REVIEW

To prepare for Section 6.2, review multiplication and division using fraction notation (Section 1.2).

Simplify.

101. $-\dfrac{2}{15} \cdot \dfrac{10}{7}$ [1.2] $-\dfrac{4}{21}$

102. $\left(\dfrac{3}{4}\right)\left(\dfrac{-20}{9}\right)$ [1.2] $-\dfrac{5}{3}$

103. $\dfrac{5}{8} \div \left(-\dfrac{1}{6}\right)$ [1.2] $-\dfrac{15}{4}$

104. $\dfrac{7}{10} \div \left(-\dfrac{8}{15}\right)$ [1.2] $-\dfrac{21}{16}$

105. $\dfrac{7}{9} - \dfrac{2}{3} \cdot \dfrac{6}{7}$ [1.2] $\dfrac{13}{63}$

106. $\dfrac{2}{3} - \left(\dfrac{3}{4}\right)^2$ [1.2] $\dfrac{5}{48}$

SYNTHESIS

TW **107.** Keith *incorrectly* simplifies

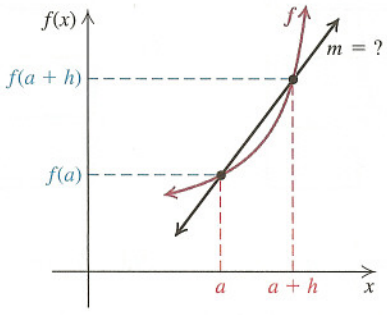
$$\frac{x^2 + x - 2}{x^2 + 3x + 2} \text{ as } \frac{x - 1}{x + 2}.$$

He then checks his simplification by evaluating both expressions for $x = 1$. Use this situation to explain why evaluating is not a foolproof check.

TW **108.** How could you convince someone that $a - b$ and $b - a$ are opposites of each other?

⊡ Answers to Exercises 77–84 are on p. IA-13.

109. Calculate the slope of the line passing through $(a, f(a))$ and $(a + h, f(a + h))$ for the function f given by $f(x) = x^2 + 5$. Be sure your answer is simplified. $2a + h$



110. Calculate the slope of the line passing through the points $(a, f(a))$ and $(a + h, f(a + h))$ for the function f given by $f(x) = 3x^2$. Be sure your answer is simplified. $6a + 3h$

111. Let

$$g(x) = \frac{2x + 3}{4x - 1}.$$

Determine each of the following.

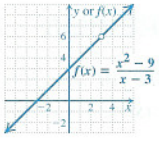
a) $g(x + h)$ ⊡

b) $g(2x - 2) \cdot g(x)$ ⊡

c) $g\left(\frac{1}{2}x + 1\right) \cdot g(x)$ ⊡

112. Graph the function given by

$$f(x) = \frac{x^2 - 9}{x - 3}.$$



(*Hint:* Determine the domain of f and simplify.)

Perform the indicated operations and simplify.

113. $\dfrac{r^2 - 4s^2}{r + 2s} \div (r + 2s)^2\left(\dfrac{2s}{r - 2s}\right)^2$ ⊡

114. $\dfrac{d^2 - d}{d^2 - 6d + 8} \cdot \dfrac{d - 2}{d^2 + 5d} \div \left(\dfrac{5d^2}{d^2 - 9d + 20}\right)^2$ ⊡

Aha! **115.** $\dfrac{6t^2 - 26t + 30}{8t^2 - 15t - 21} \cdot \dfrac{5t^2 - 9t - 15}{6t^2 - 14t - 20} \div \dfrac{5t^2 - 9t - 15}{6t^2 - 14t - 20}$ ⊡

Simplify.

116. $\dfrac{m^2 - t^2}{m^2 + t^2 + m + t + 2mt}$ $\dfrac{m - t}{m + t + 1}$

117. $\dfrac{a^3 - 2a^2 + 2a - 4}{a^3 - 2a^2 - 3a + 6}$ $\dfrac{a^2 + 2}{a^2 - 3}$

118. $\dfrac{x^3 + x^2 - y^3 - y^2}{x^2 - 2xy + y^2}$ $\dfrac{x^2 + xy + y^2 + x + y}{x - y}$

119. $\dfrac{u^6 + v^6 + 2u^3v^3}{u^3 - v^3 + u^2v - uv^2}$ $\dfrac{(u^2 - uv + v^2)^2}{u - v}$

120. $\dfrac{x^5 - x^3 + x^2 - 1 - (x^3 - 1)(x + 1)^2}{(x^2 - 1)^2}$ $-\dfrac{2x}{x - 1}$

121. Let

$$f(x) = \frac{4}{x^2 - 1} \quad \text{and} \quad g(x) = \frac{4x^2 + 8x + 4}{x^3 - 1}.$$

Find each of the following.

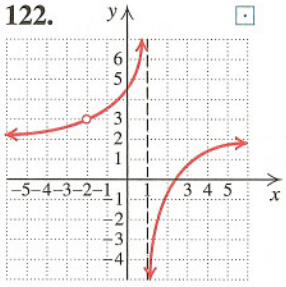
a) $(f \cdot g)(x)$ ⊡

b) $(f/g)(x)$ ⊡

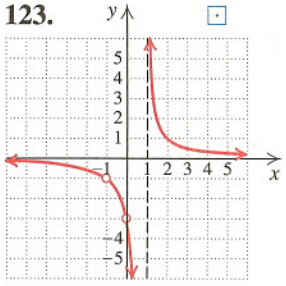
c) $(g/f)(x)$ ⊡

Determine the domain and the range of each function from its graph.

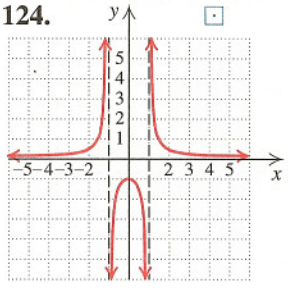
122. ⊡



123. ⊡



124. ⊡



TW **125.** Select any number x, multiply by 2, add 5, multiply by 5, subtract 25, and divide by 10. What do you get? Explain how this procedure can be used for a number trick.

Try Exercise Answers: Section 6.1

7. $\frac{40}{13}$ hr, or $3\frac{1}{13}$ hr **13.** $\{x \mid x \neq 0\}$, or $(-\infty, 0) \cup (0, \infty)$ **29.** $-\frac{4}{3}$ **33.** $g(x) = \dfrac{x - 3}{5}, x \neq -3$ **37.** $f(t) = \dfrac{t + 4}{t - 4}, t \neq 4$ **57.** $c(c - 2)$ **63.** $\dfrac{5}{x^4}$ **77.** $f(x) = \dfrac{(x + 2)(x + 4)}{x^7}, x \neq -4, 0, 2$ **89.** $x = -3$

⊡ Answers to Exercises 111, 113–115, and 121–124 are on p. IA-14.